$\left\{\frac{2np\triangle}{apm+bn-clp+cm}, \frac{2n\triangle}{apm+bn-clp+cm}, -\frac{2\triangle(lp-m)}{apm+bn-clp+cm}\right\}$ are the coordinates of C;

$\left\{0, \frac{2\triangle(qn-1)}{b(qn-1)+cmq}, \frac{2\triangle mq}{b(qn-1)+cmq}\right\}$ are the coordinates of B;

$\left\{\frac{2\triangle(qn+1)}{a(qn+1)-qcl}, 0, -\frac{2\triangle ql}{a(qn+1)-qcl}\right\}$ are the coordinates of D.

$\alpha mq+\beta pmq-\gamma p(qn-1)=0$, is the line through AB;
$\alpha nql-\beta(qmn+m-lp)+\gamma n(qn+1)=0$, is the line through CD.

Comparing, we get $q=0$ or $q=-(1/n)$, $p=m/l$ or $p=0$. There are no positive values for q when p is positive. When p is positive, $q=0$, etc.

Whatever relations we establish we cannot find p and q both real or both positive.

CALCULUS.

286. Proposed by R. D. CARMICHAEL, Princeton University.

Solve the differential equation

$$\begin{aligned}&[a_0x^3+a_1x^2y+a_2xy^2+(a_0-a_1+a_2)y^3\\&\qquad+a_3x^2+a_4xy+a_5y^2+a_6x+a_7y+a_8]dx\\&+[a_0y^3+a_1xy^2+a_2x^2y+(a_0-a_1+a_2)x^3\\&\qquad+a_3y^2+a_4xy+a_5x^2+a_6y+a_7x+a_8]dy=0.\end{aligned}$$

No solution of this problem has been received.

287. Proposed by C. N. SCHMALL, 604 East 5th Street, New York City.

An object P, being placed beyond the principal focus F of a convex lense, determine its position when its distance PQ, from its image Q, is a minimum.

Solution by G. B. M. ZERR, A. M., Ph. D., Philadelphia, Pa.

Let u=distance of object from lense, v=distance of image from lense, t=thickness of lense, and r, s=the radii of the first and second surface.

Then $\dfrac{1}{\frac{1}{v}+\frac{\mu-1}{s}}+\dfrac{1}{\frac{1}{u}-\frac{\mu-1}{r}}=\dfrac{t}{\mu}$, where μ=index of refraction, or

$$\mu\left(\frac{1}{u}+\frac{1}{v}\right)=\left(\frac{1}{r}-\frac{1}{s}\right)\mu(\mu-1)+t\left(\frac{1}{v}+\frac{\mu-1}{s}\right)\left(\frac{1}{u}-\frac{\mu-1}{r}\right)\ldots(1).$$

$$u+v+t=\text{minimum}\ldots(2).$$

$$\mu\left(\frac{du}{u^2}+\frac{dv}{v^2}\right)=t\left(\frac{1}{u}-\frac{\mu-1}{r}\right)\frac{dv}{v^2}+t\left(\frac{1}{v}+\frac{\mu-1}{s}\right)\frac{du}{u^2}.\quad du+dv=0.$$

$$\therefore\left(\mu-\frac{t(\mu-1)}{r}\right)u^2+tu=\left(\mu+\frac{t(\mu-1)}{s}\right)v^2+tv.$$

The value of v from this equation in (1) gives the value of u.
If t, the thickness of the lense, be neglected, we get $u=\pm v$.
This in $\frac{1}{u}+\frac{1}{v}=(\mu-1)\left(\frac{1}{r}-\frac{1}{s}\right)=\frac{1}{f}$, gives $\frac{2}{u}=\frac{1}{f}$, or $u=2f=v$.
Taking the formula $\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$, $u+v=$minimum, we get

$$\frac{du}{u^2}+\frac{dv}{v^2}=0.\quad du+dv=0.$$

$\therefore u=\pm v$ and $u=2f=v$, as before. The object and image are both twice the focal distance from the lense.

Also solved by C. N. Schmall.

288. **Proposed by L. H. McDONALD, M. A., Ph. D., Sometimes Tutor at Cambridge, Jersey City, N. J.**

Find $\int\frac{xdx}{(1+x^3)^{2/3}}$.

Solution by G. B. M. ZERR, A. M., Ph. D., Philadelphia, Pa.; FRANCIS RUST, M S., Pittsburg, Pa.; and V.M. SPUNAR, Pittsburg, Pa.

Let $1+x^3=x^3z^3$. Then $\frac{xdx}{(1+x^3)^{2/3}}=-\int\frac{dz}{z^3-1}=u$.

Hence, $u=-\frac{1}{3}\int\frac{dz}{z-1}+\frac{1}{3}\int\frac{(z+2)dz}{z^2+z+1}=\frac{1}{6}\log\left[\frac{z^2+z+1}{(z-1)^2}\right]+\frac{1}{\sqrt{3}}\tan^{-1}\left[\frac{2z+1}{\sqrt{3}}\right]$

$$=\frac{1}{6}\log\left[\frac{(1+x^3)^{2/3}+x(1+x^3)^{1/3}+x^2}{(1+x^3)^{2/3}-2x(1+x^3)^{1/3}+x^2}\right]$$

$$+\frac{1}{\sqrt{3}}\tan^{-1}\left[\frac{2(1+x^3)^{1/3}+x}{x\sqrt{3}}\right].$$

Also solved by J. Scheffer, S. G. Barton, and C. N. Schmall.
S. Lefsehetz and V. M. Spunar should have been credited for solving 284 and 285 in last issue.